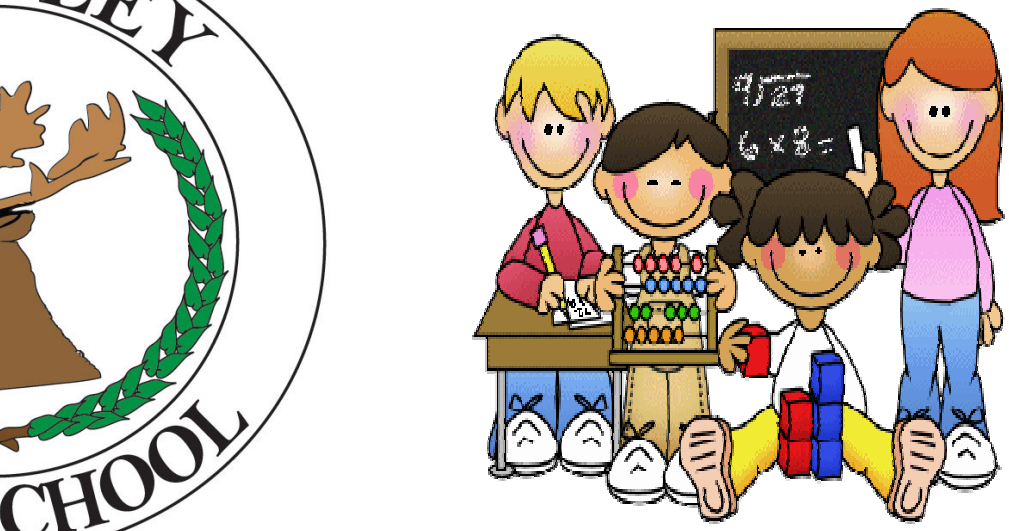
HINDLEY JUNIOR & INFANT SCHOOL



Mental to Written Calculation Policy

Pencil and paper procedures

This policy contains the key pencil and paper procedures that will be taught within our schools. It has been written to ensure consistency and progression throughout the schools.

Although the focus of the policy is on pencil and paper procedures it is important to recognise that the ability to calculate mentally lies at the heart of the Primary Framework for mathematics. The mental methods in the ***Primary Framework for teaching mathematics*** will be taught systematically from Reception onwards and pupils will be given regular opportunities to develop the necessary skills. However mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it. In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Therefore written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

During their time at this school children will be encouraged to see mathematics as both a written and spoken language. Teachers will support and guide children through the following important stages:

- developing the use of pictures and a mixture of words and symbols to represent numerical activities
- using standard symbols and conventions
- use of jottings to aid a mental strategy
- use of pencil and paper procedures
- use of a calculator

This policy concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction of pencil and paper procedures. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Therefore children will always be encouraged to look at a calculation/problem and then decide which is the best method to choose – pictures, mental calculation with or without jottings, structured recording or a calculator?

Vocabulary:

The correct terminology should be used when referring to the value of digits to support the children's understanding of place value. E.g. 68 + 47 should be read *'sixty add forty'* not *'six add four.'*

AIMS:

Children should be able to choose an efficient method, mental, written or calculator appropriate to the given task. By the end of year 6, children working at Level 4 and above will have been taught, and be secure with, a compact standard method for each operation.

General Progression: (see national strategy guidance paper for calculation)

- Establish mental methods, based on a good understanding of place value
- Use of informal jottings to aid mental calculations
- Develop use of empty number line to help mental imagery and aid recording
- Use partitioning and recombining to aid informal methods
- Introduce expanded written methods
- Develop expanded methods into compact standard written form

Before carrying out a calculation, children will be encouraged to consider:

- **Can I do it in my head? (using rounding, adjustment)**
- **The size of an approximate answer (estimation)**
- **Could I use jottings to keep track of the calculation?**
- **Do I need to use an expanded or compact written method?**

When are children ready for written calculations?

Addition and subtraction:

- Do they know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Multiplication and Division:

- Do they know the 2,3,4,5 and 10 times tables?
- Do they know the result of multiplying by 1 and 0?
- Do they understand 0 as a place holder?
- Can they multiply two and three digit numbers by 10 and 100?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?
- These lists are not exhaustive but are a guide for the teacher to judge when a child is ready to move from informal to formal methods of calculation.

ADDITION

Stage 1

+ = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

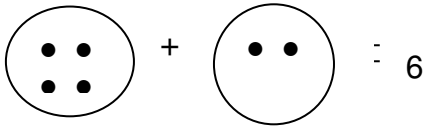
2 = 1+ 1
2 + 3 = 4 + 1
3 = 3
2 + 2 + 2 = 4 + 2

Missing numbers need to be placed in all possible places.

3 + 4 = □	□ = 3 + 4
3 + □ = 7	7 = □ + 4
□ + 4 = 7	7 = 3 + □
□ + □ = 7	7 = □ + □

Activities

Children should have access to a wide range of counting equipment, everyday objects, as well as hoops, sorting trays, number tracks and numbered number lines.



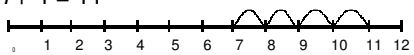
Teacher modelling

Drawing jumps on numbered number lines to support understanding of the mental method

Children

To create their own jumps using rulers, fingers, pens, bodies etc.

7+ 4 = 11



Stage 2

+ = signs and missing numbers

Continue using a range of equations as in Stage1 but with appropriate, larger numbers.

Extend to -

14 + 5 = 10 + □
32 + □ + □ = 100
35 = 1 + 5 + □

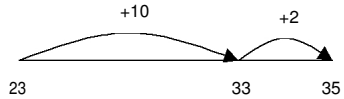
Least significant number first

Partition into tens and ones and recombine for written

23 + 12 =
3 + 2 =5
20 + 10 = 30
5 + 30 = 35

Count on in tens and ones

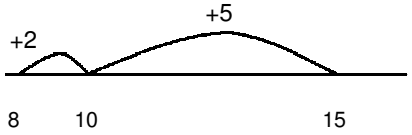
23 + 12 = 23 + 10 + 2
33 + 2 = 35



Partitioning and bridging through 10.

The steps in addition often bridge through a multiple **of 10** e.g. children should be able to partition the 7 to relate adding the 2 and then the 5.

8 + 7 = 15

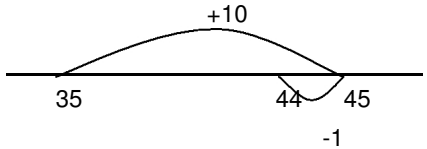


Add 9 or 11 by adding 10 and adjusting by 1

e.g.

Add 9 by adding 10 and adjusting by 1

35 + 9 = 44



Stage 3

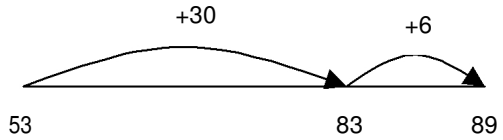
+ = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate, larger numbers.

Partition into tens and ones

- Partition both numbers and recombine.
- Count on by partitioning the lowest number only e.g.

36 + 53 = 53 + 30 + 6
= 83 + 6
= 89

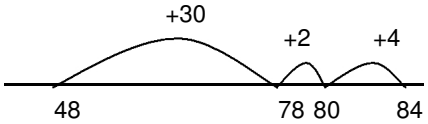


Add a near multiple of 10 to a two-digit number

Secure mental methods by using a number line to model the method. Continue as in Stages 2 but with appropriate numbers e.g. 35 + 19 is the same as 35 + 20 – 1.

Children need to be secure adding multiples of 10 to any two-digit number including those that are not multiples of 10.

48 + 36 = 84



Pencil and paper procedures

83 + 42 = 125

either ***or***

1. Vertical expansion

83
+ 42
5
120
125

2. Horizontal expansion

80 + 3
+ 40 + 2
120 + 5 = 125

Addition

Stage 4

+ = signs and missing numbers

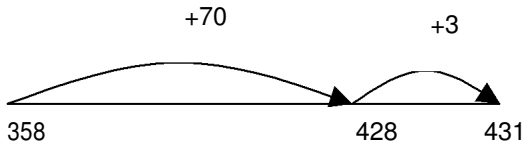
Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

Partition (the lowest number) into hundreds, tens and ones and recombine

Either partition both numbers and recombine or partition the second number only e.g.

358 + 73 = 358 + 70 + 3

428 + 3 = 431



Add the nearest multiple of 10, then adjust

Continue as in Stages 2 and 3 but with appropriate numbers e.g. 63 + 29 is the same as 63 + 30 - 1

Pencil and paper procedures

367 + 185 = 552

300 + 60 + 7
100 + 80 + 5
400 +140 +12 = 552

Extend to decimals in the context of money.

Stage 5

+ = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

Add or subtract the nearest multiple of 10 or 100, then adjust

Continue as in Stages 2, 3 and 4 but with appropriate numbers e.g. 458 + 79 = is the same as 458 + 80 - 1

Pencil and paper procedures

Extend to numbers with at least four digits 3587 + 675 = 4262

```
  367
+ 185
  552
  11
```

```
  3587
+  675
  4262
  111
```

Revert to expanded methods if the children experience any difficulty.

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

```
  72.8
+54.6
 127.4
 1  1
```

Stage 6

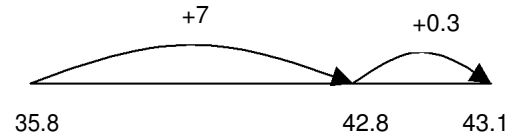
+ = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

Partition into hundreds, tens, ones and decimal fractions and recombine

Either partition both numbers and recombine or partition the second number only e.g.

35.8 + 7.3 = 35.8 + 7 +0.3
42.8 + 0.3 = 43.1



Add the nearest multiple of 10, 100 or 1000, then adjust

Continue as in Stages 2, 3, 4 and 5 but with appropriate numbers including extending to adding 0.9, 1.9, 2.9 etc

Pencil and paper procedures

Extend to numbers with any number of digits and decimals with 1, 2 and/or 3 decimal places.

13.86 + 9.481 = 23.341

```
  13.86
+  9.481
  23.341
  11 1
```

Revert to expanded methods if the children experience any difficulty.

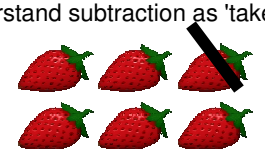
Subtraction

Stage 1

- = signs and missing numbers

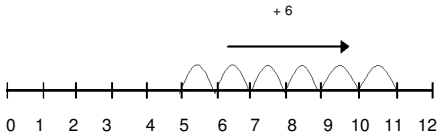
7 - 3 = □ □ = 7 - 3
7 - □ = 4 4 = □ - 3
□ - 3 = 4 4 = 7 - □
□ - = 4 4 = □ - □

- Understand subtraction as 'take away'



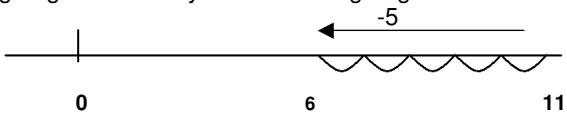
- Find a 'difference' by counting up;

I have saved 5p. The socks that I want to buy cost 11p. How much more do I need in order to buy the socks?



Use practical and informal written methods to support the subtraction of a one-digit number from a one digit or two-digit number and a multiple of 10 from a two-digit number.

I have 11 toy cars. There are 5 cars too many to fit in the garage. How many cars fit in the garage?



Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences
Recording by
- drawing jumps on prepared lines
- constructing own lines

Stage 2

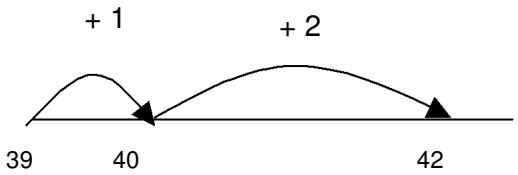
- = signs and missing numbers

Continue using a range of equations as in Stages 1 but with appropriate numbers.

Extend to 14 + 5 = 20 - □

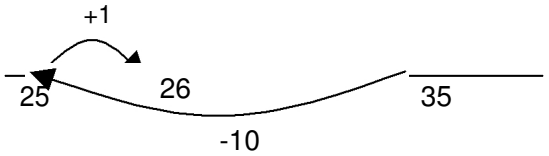
Find a small difference by counting up

42 – 39 = 3



Subtract 9 or 11. Begin to add/subtract 19 or 21

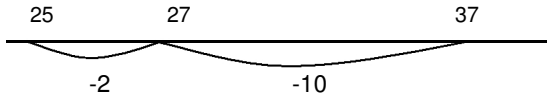
35 – 9 = 26 9 (take 10 add 1) 11(take 10 take 1)



Use known number facts and place value to subtract

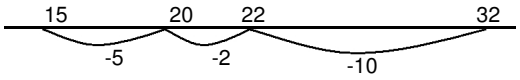
(partition second number only)

37 – 12 = 37 – 10 – 2
= 27 –2
= 25



Bridge through 10 where necessary

32 - 17



Stage 3

- = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

Find a small difference by counting up

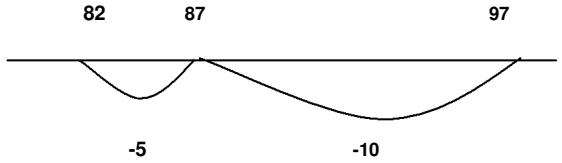
Continue as in Stages 2 but with appropriate numbers
e.g. 102 – 97 = 5

Subtract mentally a 'near multiple of 10' to or from a two-digit number

Continue as in Stages 2 but with appropriate numbers
e.g. 78 – 49 is the same as 78 – 50 + 1

Use known number facts and place value to subtract

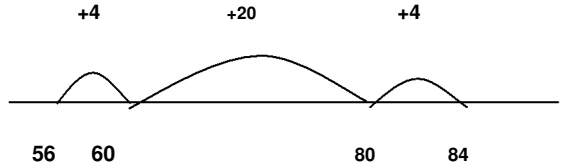
Continue as in Stages 2 but with appropriate numbers
e.g. 97 – 15 = 82



With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations s**uch as 57 – 12, 86 – 77 or 43 – 28.**

Pencil and paper procedures (inverse) 56 +28 = 84

Complementary addition
84 – 56 = 28



Subtraction

Stage 4

- = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

Find a small difference by counting up

e.g. 5003 – 4996 = 7

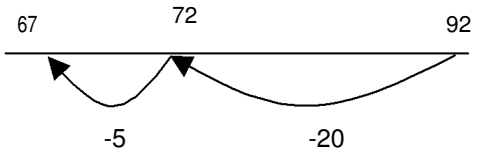
This can be modelled on an empty number line (see complementary addition below). Children should be encouraged to use known number facts to reduce the number of steps.

Subtract the nearest multiple of 10, then adjust.

Continue as in Stages 2 and 3 but with appropriate numbers.

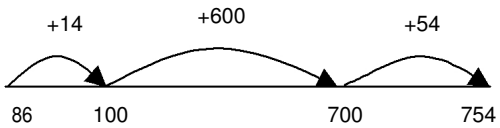
Use known number facts and place value to subtract

92 – 25 = 67



Pencil and paper procedures

Complementary addition 754 – 86 = 668



For those children with a secure mental image of the number line they could record the jumps only:
754 – 86 = 668

Partition

93 – 76 =
93 – 70 = 23
23 – 6 = 17

Stage 5

- = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

Find a difference by counting up

e.g. 8006 – 2993 = 5013

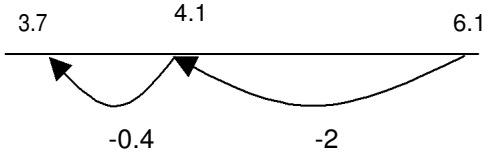
This can be modelled on an empty number line (see complementary addition below).

Subtract the nearest multiple of 10 or 100, then adjust.

Continue as in Stages 2, 3 and 4 but with appropriate numbers.

Use known number facts and place value to subtract

6.1 – 2.4 = 3.7



Partitioning or expanded decomposition

767 – 619 =

700 + 60 + 7
- 600 + 10 + 9

→

700 + 50 + 17
- 600 + 10 + 9

Extend to 2 places of decimals

Stage 6

- = signs and missing numbers

Continue using a range of equations with appropriate numbers.

Find a difference by counting up

e.g. 8000 – 2785 = 5215

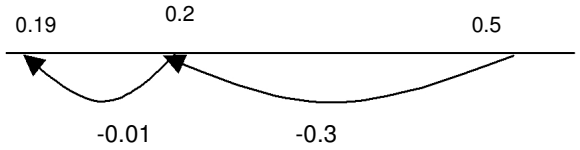
To make this method more efficient, the number of steps should be reduced to a minimum through children knowing:

Complements to 1, involving decimals to two decimal places (0.16 + 0.84)
Complements to 10, 100 and 100

Subtract the nearest multiple of 10, 100 or 1000, then adjust

Continue as in Stages 2, 3, 4 and 5 but with appropriate numbers.

Use known number facts and place value to subtract 0.5 – 0.31 = 0.19



Partitioning or compact decomposition

```
  5 13
 3 6 3
-1 2 7
 2 3 6
```

Extend to 2 places of decimals

Multiplication

Stage 1

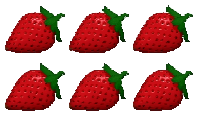
Multiplication is related to doubling and counting groups of the same size.

Counting using a variety of practical resources

Counting in 2s e.g. counting socks, shoes, animal's legs…
Counting in 5s e.g. counting fingers, fingers in gloves, toes…
Counting in 10s e.g. fingers, toes…

Pictures / marks

There are 3 sweets in one bag.
How many sweets are there in 5 bags?



Lots of

Put objects into groups

Looking at columns
2 + 2 + 2
3 groups of 2

Looking at rows
3 + 3
2 groups of 3

Stage 2

x = signs and missing numbers

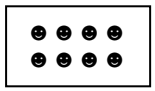
7 x 2 =☐ ☐= 2 x 7

7 x☐ = 14 14 = ☐ x 7

☐x 2 = 14 14 = 2 x ☐

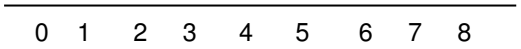
☐ x ☐ = 14 14 = ☐ x ☐

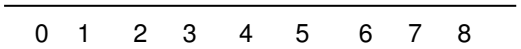
Arrays and repeated addition



4 x 2 or 4 + 4

2 x 4 or 2 + 2 + 2 + 2





Doubling multiples of 5 up to 50

15 x 2 = 30

Partition

Children need to be secure with partitioning numbers into 10s and 1s

Double 15

10 + 5

20 + 10 = 30

Stage 3

x = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers. Know by heart x2, x3, x4, x5 and x10. Multiply a single digit by 1, 10, 100

Arrays and repeated addition

Continue to understand multiplication as repeated addition and continue to use arrays (as in Stage 2).

Derive related facts (4 for price of one)

7 x 5 = 35
5 X 7 = 35
35 ÷ 7 = 5
35 ÷ 5 = 7

Doubling multiples of 5 up to 50

35 x 2 = 70

30 5
↓ ↓
60 + 10 =70

Use known facts and place value to carry out simple multiplications

Use the same method as above (partitioning), e.g.

32 x 3 = 96

30 x 3 = 90

2 x 3 = 6

90+6 = 96

Multiplication

Stage 4

X = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers. Know by heart multiplication to 12 x 12.

Partitioning

18 x 9 = 162

18 x 9 = (10 x 9) + (8 x 9) = 162

OR

Use the grid method of multiplication (as below)

Pencil and paper procedures

Grid method
23 x 7 is approximately 20 x 10 = 200

x	20	3
7	140	21

140
+ 21
161

Stage 5

x = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers.

Partitioning
47 x 6 = 282

47 x 6 = (40 x 6) + (7 x 6) = 282

OR

Use the grid method of multiplication (as below)

Pencil and paper procedures
Grid method
72 x 38 is approximately 70 x 40 = 2800

x	70	2
30	2100	60
8	560	16
	2660	+ 76

2660
+ 76
2736
1

Extend to simple decimals with one decimal place.

x	80	3	0.4
20	1600	60	8.0
6	480	18	2.4
	2080	+ 78	+ 10.4

2080.0
78.0
+ 10.4
2168.4

Stage 6

x = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers

Partitioning
87 x 6 = 522

87 x 6 = (80 x 6) + (7 x 6) = 522

OR

Use expanded form leading to long multiplication.

Pencil and paper procedures

Extended form

246
x 7
1400
280
42
1722

Leading to long multiplication

352
x 27
7040
2464
9504

Extend to decimals with up to two decimal places.

4.62
x 3
13.86
1

Division

Stage 1

Sharing

Requires secure counting skills
-see counting and understanding number strand
Develops importance of one-to-one correspondence

Practical activities involving sharing, distributing cards when playing a game, putting objects onto plates, into cups, hoops etc.

Grouping

Sorting objects into 2s / 3s/ 4s etc
How many pairs of socks are there?

There are 12 crocus bulbs. Plant 3 bulbs in each pot. How many pots are needed?
Jo has 12 Lego wheels. How many cars can she make?

Stage 2

÷ = signs and missing numbers

6 ÷ 2 =□ □= 6 ÷ 2
6 ÷ □ = 3 3 = 6 ÷□
□÷ 2 = 3 □ = □ ÷ 2
□÷ □ = 3 3 = □ ÷ □

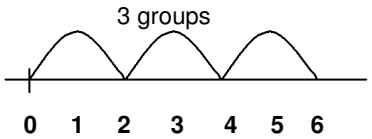
Understand division as sharing and grouping

6 ÷ 2 can be modelled as:
Sharing- 6 shared between 2 (see stage 1 diagram)
OR

Grouping

Link to counting and understanding number strand Count up to 100 objects by grouping them and counting in tens, fives or twos;…
Find one half, one quarter and three quarters of shapes and sets of objects
6 ÷ 2 can be modelled as:
There are 6 strawberries
How many people can have 2 each? How many 2s make 6?

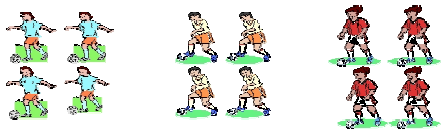
6 ÷ 2 can be modelled as:



In the context of money count forwards and backwards using 2p, 5p and 10p coins

Practical grouping e.g. in PE

12 children get into teams of 4 to play a game. How many teams are there?



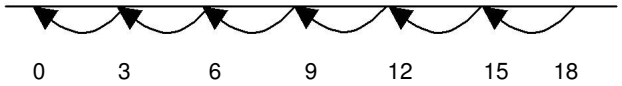
Stage 3

÷ = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers.

Understand division as sharing and grouping

18 ÷ 3 can be modelled as:
Sharing – 18 shared between 3 (see Stage 1 diagram)
OR
Grouping - How many 3's make 18?



Repeated subtraction (minus 3 minus 3 etc)

Remainders

16 ÷ 3 = 5 r1
Sharing - 16 shared between 3, how many left over?
Grouping – How many 3's make 16, how many left over? e.g.

Division

Stage 4

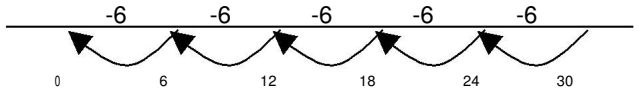
÷ = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers.

Sharing and grouping

30 ÷ 6 can be modelled as:

grouping – groups of 6 placed on no. line and the number of groups counted e.g.



sharing – sharing among 6, the number given to each person

Remainders

41 ÷ 4 = 10 r1

Menu (to support subtraction of groups):

1 x 4 = 4 2 x 4 = 8

10 x 4 = 40 5 x 4 = 20

Pencil and paper procedures

72 ÷ 5

'chunking'

72
- 50 (10 x 5)
22
- 20 (4 x 5)
2

Answer : 14 remainder 2

Stage 5

÷ = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers.

Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

Remainders

Quotients expressed as fractions or decimal fractions 41 ÷ 4 = 10 ¼ or 10.25

Pencil and paper procedures

Continue with efficient chunking method OR
Short division (HTU ÷ U)

196 ÷ 6 is approximately just over 180 ÷ 6 = 30

196 ÷ 6

Before starting with Chunking, ask children to record multiplication facts to help with division

23 R 8 or 8/24 = 1/3

24) 560
-480
80
- 72
8

10 x 24 = 240
20 x 24 = 480
5 x 24 = 120

Stage 6

÷ = signs and missing numbers

Continue using a range of equations as in Stages 2 but with appropriate numbers.

Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

Remainders

Quotients expressed as fractions or decimal fractions
676 ÷ 8 = 84.5

Short Division

642 ÷ 3 =

2 1 4
3 | 6 4 12

3 into 6 = 2 (no remainder)

3 into 4 = 1 remainder 1

1 makes 2 into 12, so:

3 into 12 = 4

642 ÷ 3 = 214

Pencil and paper procedures

Continue with efficient chunking method OR
Long division (HTU ÷ TU)

560 ÷ 24. The answer lies between 24 x 20 = 480 and 24 x 30 = 720

3 2 r. 4 or 4/6 = 2/3

6) 1 9 1 6